ECS 455 Chapter 1

Introduction & Review

Radio

1.3 Wireless Channel (Part 1)

noise, interference } random, time-varying

Office Hours:
BKD 3601-7

Tuesday	**9:30-10:30**
Tuesday	**13:30-14:30**
Thursday	**13:30-14:30**

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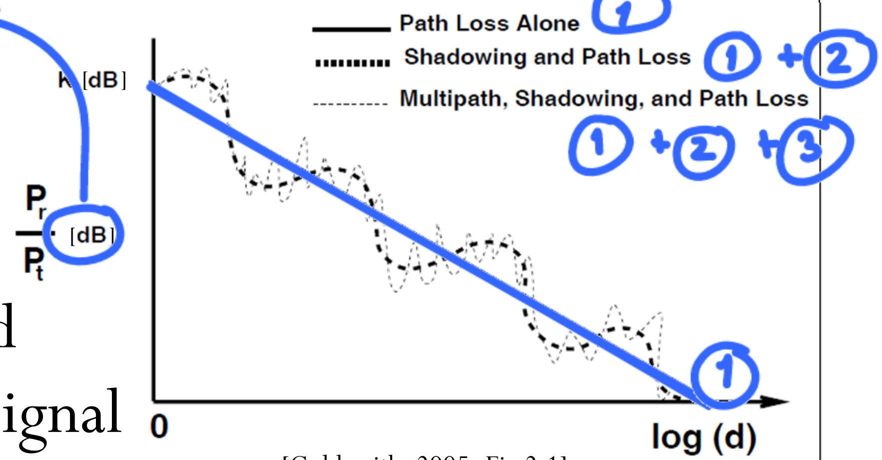
Wireless Channel

- **Large-scale** propagation effects
  1. Path loss
  2. Shadowing
- **Small-scale** propagation effects
  - Variation due to the constructive and destructive addition of **multipath** signal components.
  - Occur over very short distances, on the order of the signal wavelength.

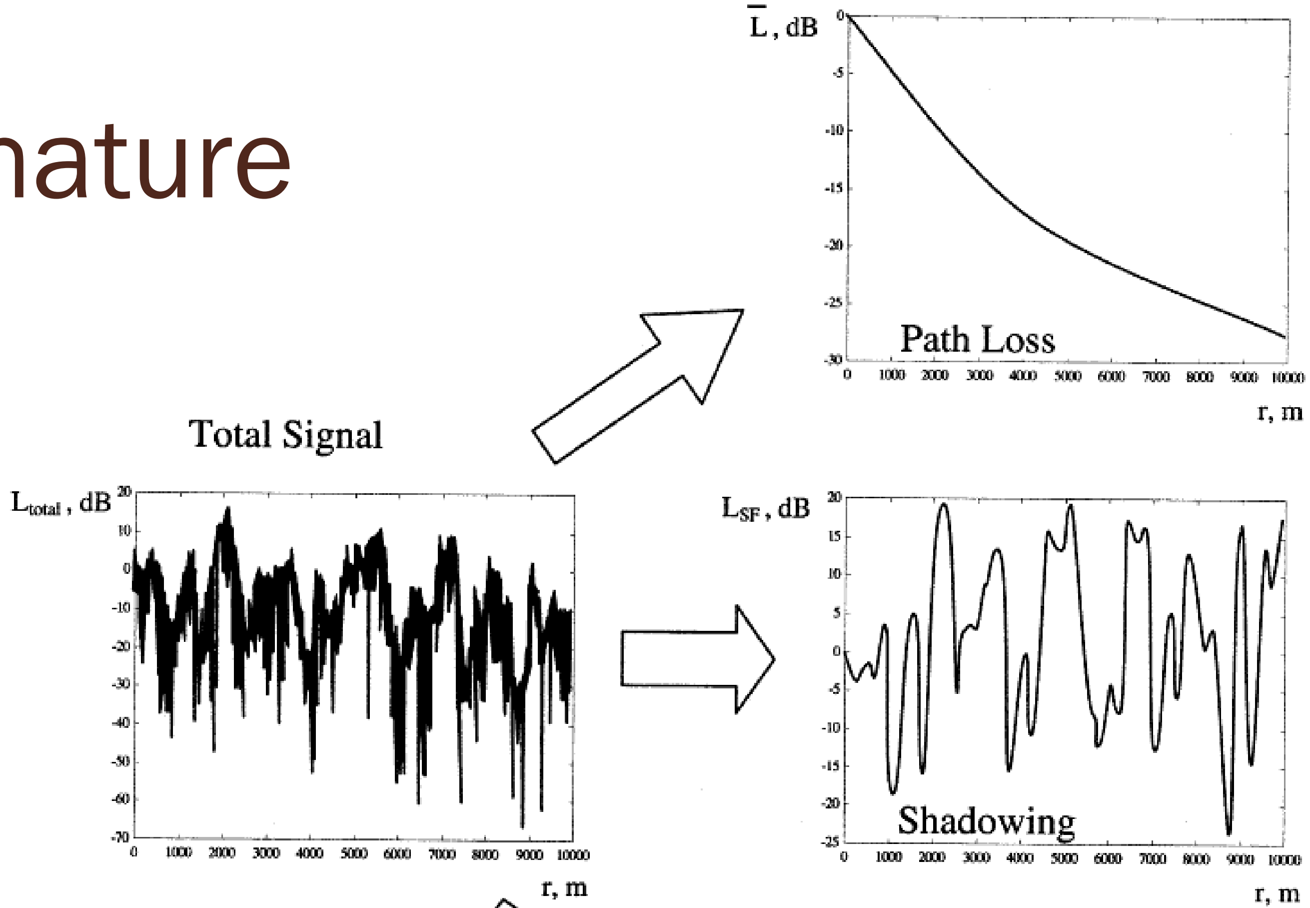


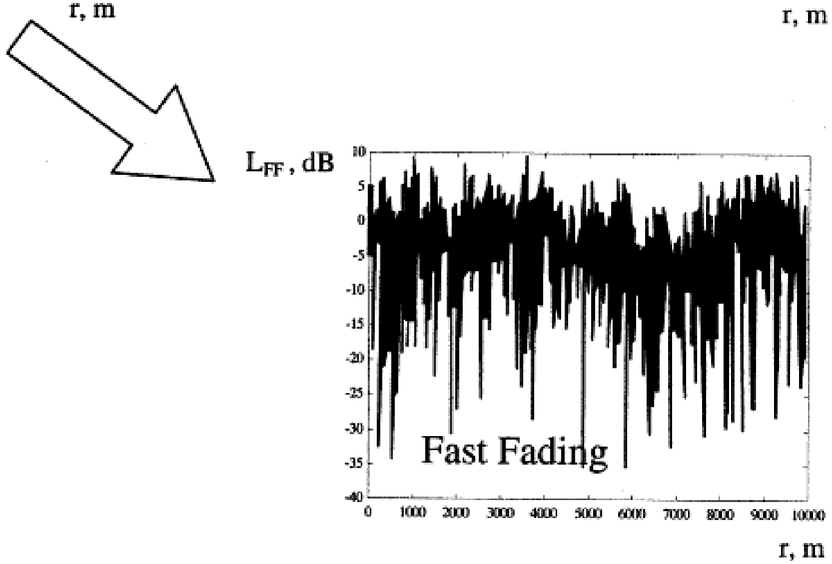
[Goldsmith, 2005, Fig 2.1]

$$\lambda = \frac{c}{f}, \quad c \approx 3\times10^{8}\ [\text{m/s}]$$

$f = 3$ GHz → $\lambda = 0.1$ m

Triple nature



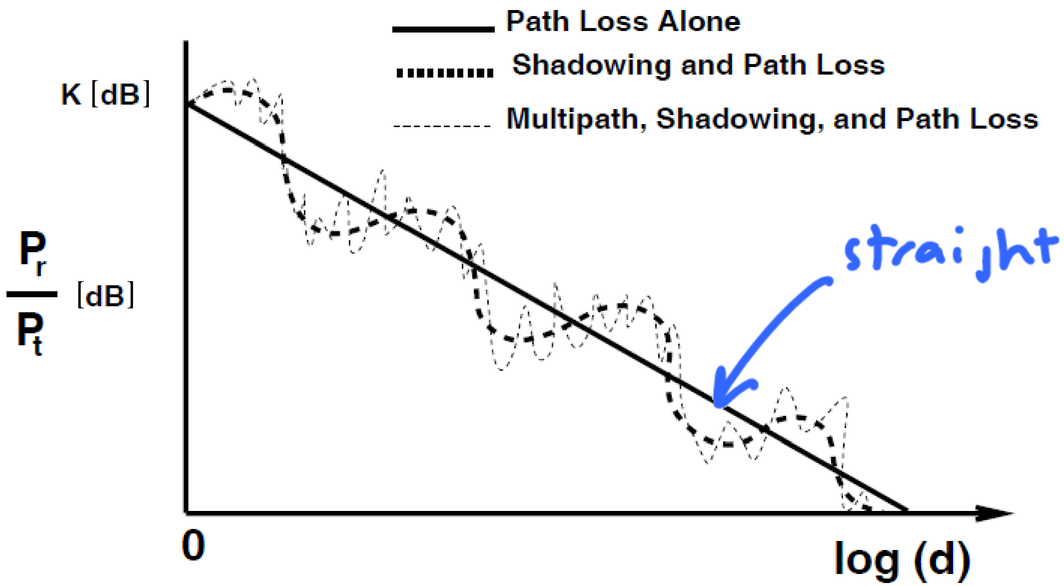


Variations of signal level have a triple nature.

[Blaunstein, 2004, Fig 12.4]

① Path loss

- Caused by
  - dissipation of the power radiated by the transmitter
  - effects of the propagation channel
- Models generally assume that it is the same at a given transmit-receive distance.
- Variation occurs over **large distances** (100-1000 m)



straight line in log-log plot

[Goldsmith, 2005, Fig 2.1]

Path Loss (PL)

$$P_L = \frac{\text{Transmitted power}}{\text{Average received power}} = \frac{P_t}{P_r}$$

Averaged over any random variations

- **Free-Space** Path Loss Model:

$$\frac{P_r}{P_t} \propto \frac{1}{d^2}$$

  - P_r falls off inversely proportional to the square of the distance d between the Tx and Rx antennas.

- **Simplified** Path Loss Model:

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

To be discussed

(Path loss of the free-space model)

Friis Equation (Free-Space PL)

- One of the most fundamental equations in antenna theory

1 for non-directional antennas

$$\frac{P_r}{P_t} = \left(\frac{\sqrt{G_{Tx} G_{Rx}}\lambda}{4\pi d} \right)^2 = \left(\frac{\sqrt{G_{Tx} G_{Rx}}c}{4\pi df} \right)^2 \propto \frac{1}{d^2 f^2}$$

- Lose more power at higher frequencies.

0.7 GHz → 2.4 GHz → 5 GHz → 60 GHz

10.7 dB loss	6.4 dB loss	21.6 dB loss
$20\log_{10}\frac{2.4}{0.7}$	$20\log_{10}\frac{5}{2.4}$	$20\log_{10}\frac{60}{5}$

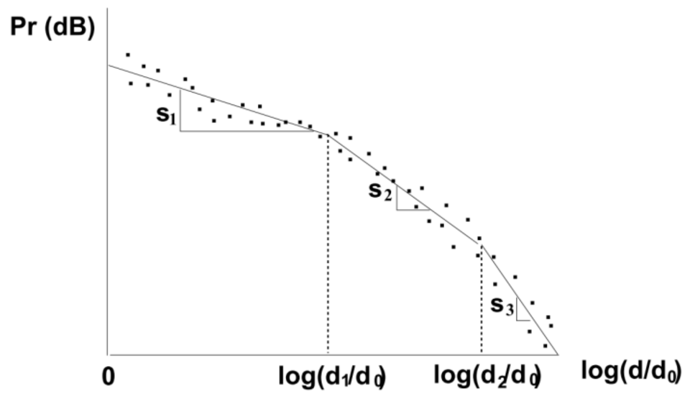
- Some of these losses can be offset by reducing the maximum operating range.
  - The remaining loss must be compensated for by increasing the antenna gain.

More Path Loss Models

- Analytical models
  - Maxwell's equations
  - Ray tracing

} Prohibitive (complex, impractical) Need to know/specify "almost everything" about the environment.

- Empirical models: Developed to predict path loss in typical environment.
  - Okumura
  - Hata
  - COST 231
    - by EURO-COST (EUROpean COoperative for Scientific and Technical research)
  - Piecewise Linear (Multi-Slope) Model
- Tradeoff: Simplified Path Loss Model



Simplified Path Loss Model

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

$$\underset{[\text{dB}]}{10\log_{10}} \frac{P_r}{P_t} = \left(10\log_{10} K d_0^{\gamma}\right) - 10\gamma \log_{10} d$$

Captures the essence of signal propagation without resorting to complicated path loss models, which are only approximations to the real channel anyway!

- K is a unitless constant which depends on the antenna characteristics and the average channel attenuation
  - $\left(\frac{\lambda}{4\pi d_0}\right)^2$ for free-space path gain at distance d_0 assuming omnidirectional antennas
- d_0 is a reference distance for the antenna far-field
  - Typically 1-10 m indoors and 10-100 m outdoors.

(Near-field has scattering phenomena.)

- γ is the **path loss exponent**.

[Goldsmith, 2005, Table 2.2]

Path Loss Exponent γ

- 2 in free-space model
- 4 in two-ray model
  [Goldsmith, 2005, eq. 2.17]
- Cellular: 3.5 – 4.5
  [Myung and Goodman, 2008 , p 17]
- Larger @ higher freq.
- Lower @ higher antenna heights

Environment	γ range
Urban macrocells	3.7-6.5
Urban microcells	2.7-3.5
Office Building (same floor)	1.6-3.5
Office Building (multiple floors)	2-6
Store	1.8-2.2
Factory	1.6-3.3
Home	3

Indoor Attenuation Factors

- Building penetration loss: 8-20 dB (better if behind windows)
- Attenuation between floors
  - @ 900 MHz
    - 10-20 dB when the Tx and Rx are separated by a single floor
    - 6-10 dB per floor for the next three subsequent floors
    - A few dB per floor for more than four floors
  - Typically worse at higher frequency.
- Attenuation across floors

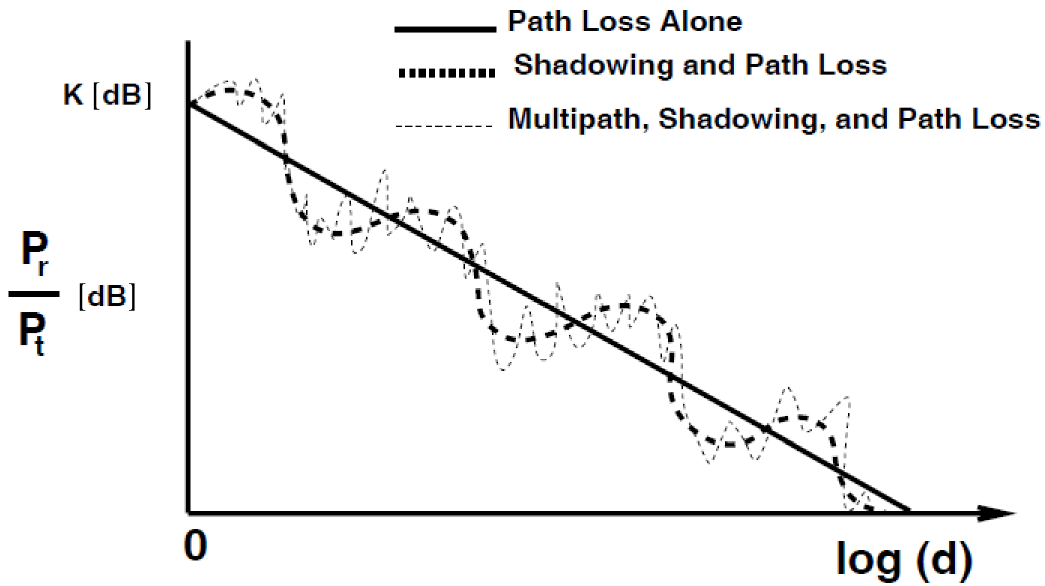
Partition Type	Partition Loss in dB
Cloth Partition	1.4
Double Plasterboard Wall	3.4
Foil Insulation	3.9
Concrete wall	13
Aluminum Siding	20.4
All Metal	26

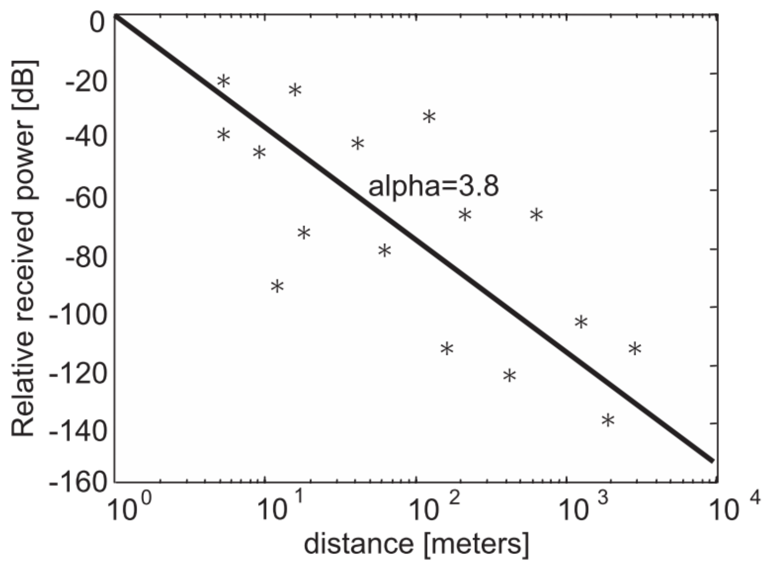
[Goldsmith, 2005, Sec. 2.5.5]

Shadowing (or Shadow Fading)

- Additional attenuation caused by **obstacles** (**large objects** such as buildings and hills) between the transmitter and receiver.
  - Think: cloud blocking sunlight
- Attenuate signal power through absorption, reflection, scattering, and diffraction.
- Variation occurs over distances proportional to the length of the obstructing object (**10-100 m** in outdoor environments and less in indoor environments).

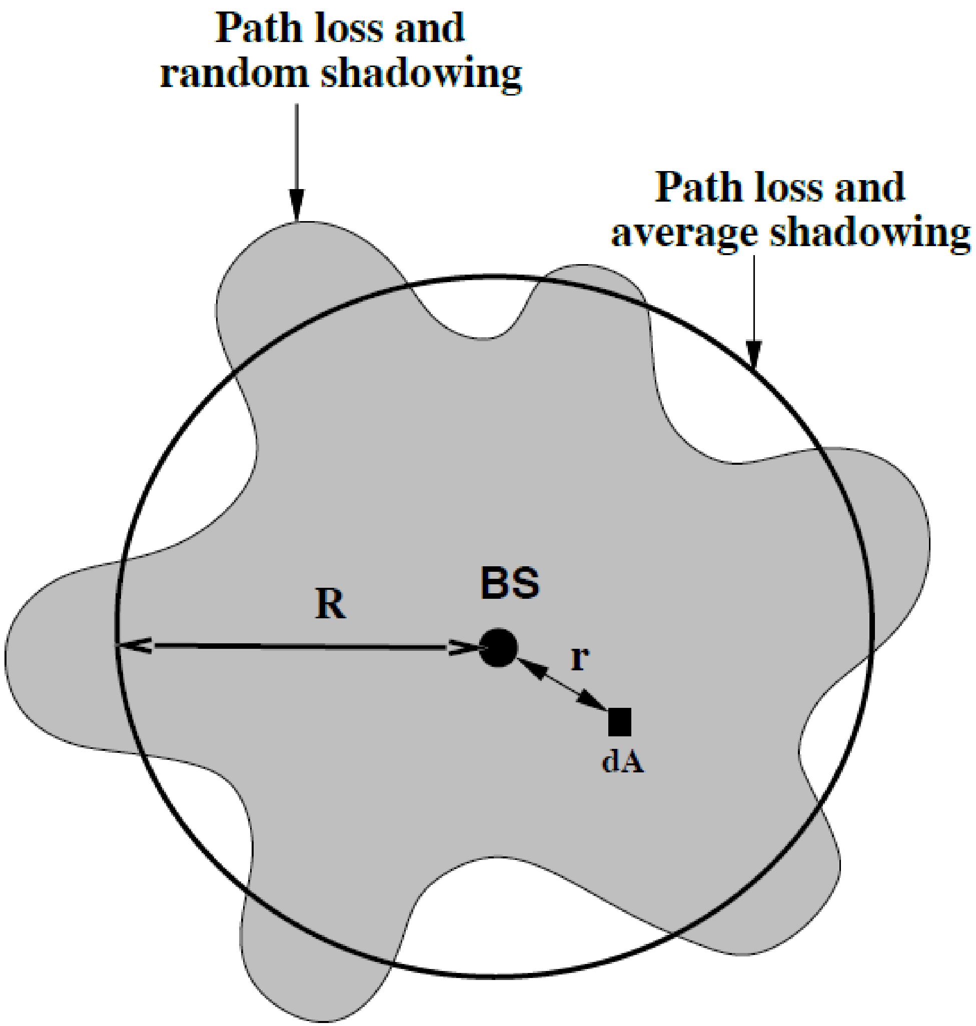
[Goldsmith, 2005, Fig 2.1]





[Myung and Goodman, 2008, Fig 2.1]

Contours of Constant Received Power



[Goldsmith, 2005, Fig 2.10]

Log-normal shadowing

- Random variation due to blockage from objects in the signal path and changes in reflecting surfaces and scattering objects → random variations of the received power at a given distance

$$10\log_{10}\frac{P_t}{P_r} \sim \mathcal{N}\left(\mu, \sigma^2\right)$$

σ: 4 – 13 dB with higher values in urban areas and lower ones in flat rural environments.

μ, σ: in dB

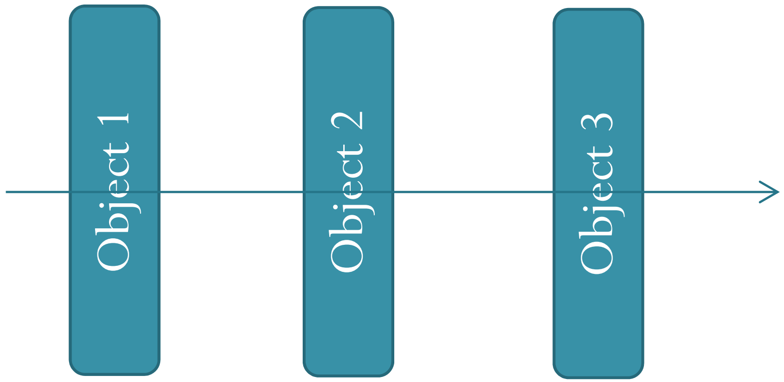
- This model has been confirmed empirically to accurately model the variation in received power in both outdoor and indoor radio propagation environments.

[Erceg et al, 1999] and [Ghassemzadeh et al, 2003]

Log-normal shadowing (motivation)

- Location, size, dielectric properties of the blocking objects as well as the changes in reflecting surfaces and scattering objects that cause the random attenuation are generally unknown

  $\Rightarrow$ statistical models must be used to characterize this attenuation.
- Assume a large number of shadowing objects between the transmitter and receiver



Without the objects, the attenuation factor is $K\left(\frac{d_0}{d}\right)^{\gamma}$.

Each object introduce extra power loss factor of α_i.

So,

$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}\prod_i \alpha_i$$

$$10\log_{10}\frac{P_r}{P_t} = 10\log_{10} K\left(\frac{d_0}{d}\right)^{\gamma} + \sum_i 10\log_{10}\alpha_i$$

By CLT, this is approximately Gaussian

$3 + \mathcal{N}(5, 2^2) = \mathcal{N}(8, 2^2)$

PDF of Lognormal RV

- Consider a random variable

$$R = \frac{P_t}{P_r}$$

- Suppose

$$10\log_{10} R \sim \mathcal{N}\left(\mu, \sigma^2\right)$$

Here, it should be clear why the unit of σ is in dB.

- Then,

$$f_R(r) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} \frac{10}{\ln 10} \frac{1}{r} e^{-\frac{1}{2}\left(\frac{(10\log r)-\mu}{\sigma}\right)^2}, & r > 0 \\ 0, & \text{otherwise.} \end{cases}$$

For typical cellular environment, σ is in the range of 5-12 dB. [Proakis and Salehi, 2007, p 843]

PDF of Lognormal RV (Proof)

Suppose $c\log_b Y \sim \mathcal{N}\left(\mu,\sigma^2\right)$.

Let $X = c\log_b Y$. Note that $X = c\log_b Y = \frac{c}{\ln b}\ln\left(Y\right) = k\ln\left(Y\right)$.

Then, $Y = e^{\frac{X}{k}}$ where $k = \frac{c}{\ln b}$.

Recall, from ECS315 that to find the pdf of $Y = g(X)$ from the pdf of X, we first find the cdf of Y and then differentiate to get its pdf:

$$F_Y\left(y\right) = P\left[Y \le y\right] = P\left[e^{\frac{X}{k}} \le y\right] = P\left[X \le k\ln\left(y\right)\right] = F_X\left(k\ln\left(y\right)\right).$$

$$f_Y\left(y\right) = \frac{d}{dy}F_X\left(k\ln\left(y\right)\right) = \frac{k}{y}f_X\left(k\ln\left(y\right)\right) = \frac{1}{\sqrt{2\pi}\sigma}\frac{k}{y}e^{-\frac{1}{2}\left(\frac{k\ln\left(y\right)-\mu}{\sigma}\right)^2}.$$

PDF of Lognormal RV (Proof)

Suppose $c\log_b Y \sim \mathcal{N}\left(\mu,\sigma^2\right)$.

Let $X = c\log_b Y$. Note that $X = c\log_b Y = \dfrac{c}{\ln b}\ln\left(Y\right) = k\ln\left(Y\right)$.

Then, $Y = e^{\frac{X}{k}}$ where $k = \dfrac{c}{\ln b}$.

Alternatively, to find the pdf of $Y = g(X)$ from the pdf of X, when g is monotone, we may use the formula:

$$f_X\left(x\right)\left|dx\right| = f_Y\left(y\right)\left|dy\right| \Longrightarrow \boxed{f_Y\left(y\right) = \left|\frac{dx}{dy}\right| f_X\left(x\right)}$$

This gives $f_Y\left(y\right) = \dfrac{k}{y} f_X\left(c\log_b y\right)$ (same as what we found earlier).

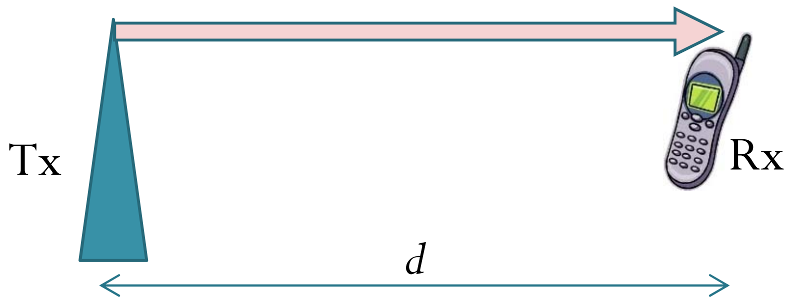
Ray tracing (a prelude)

- Approximate the solution of Maxwell's equations
  - Approximate the propagation of electromagnetic waves by representing the wavefronts as simple **particles**.
  - Thus, the reflection, diffraction, and scattering effects on the wavefront are approximated using **simple geometric equations** instead of Maxwell's more complex wave equations.
- Assumption: the received waveform can be approximated by the sum of the free space wave from the transmitter plus the reflected free space waves from each of the reflecting obstacles.

$$x(t) = \sqrt{2P_t}\cos(2\pi f_c t)$$

$$y(t) = ?$$

$\bigcirc \cos\left(2\pi f_c\left(t - \frac{d}{c}\right)\right)$



Energy and Power

$|z|^2 = z z^*$

instantaneous power $p(t) = \frac{v^2(t)}{R} = i^2(t) R$

$R = 1$

- Consider a signal $g(t)$.
- Total (normalized) **energy**:

Parseval's Theorem

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \lim_{T\to\infty} \int_{-T}^{T} |g(t)|^2 \, dt = \int_{-\infty}^{\infty} |G(f)|^2 df.$$

$$\Psi_g(f) = |G(f)|^2$$

ESD: Energy Spectral Density

$$\langle g, g \rangle = \int_{-\infty}^{\infty} g(t)\, g^*(t)\, dt = \int_{-\infty}^{\infty} G(f)\, G^*(f)\, df$$

- Average (normalized) **power**:

$$P_g = \left\langle |g(t)|^2 \right\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |g(t)|^2 \, dt.$$

time average

$$\langle x(t) \rangle = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x(t)\, dt$$

Time average operator:

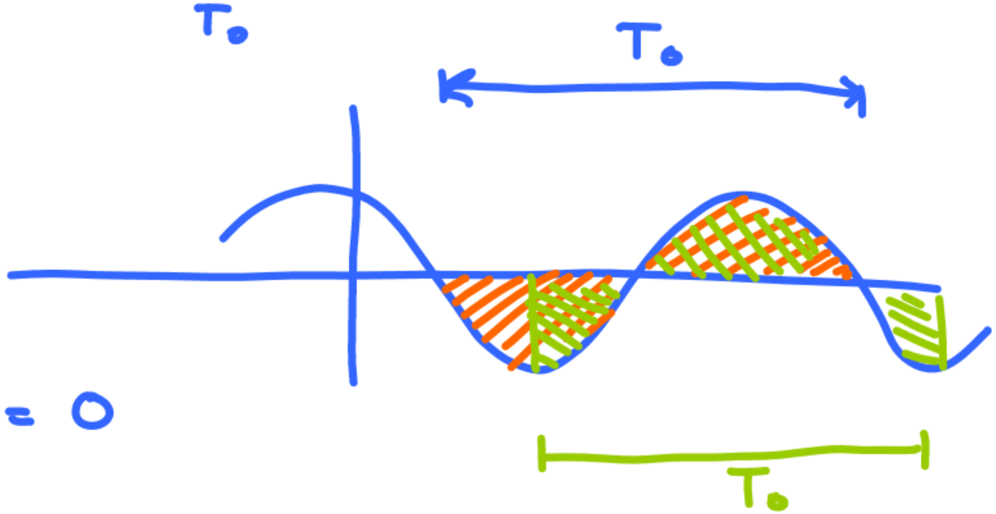
Fact: Suppose $x(t)$ is periodic with period T_0.

Then, $\langle x(t) \rangle = \frac{1}{T_0} \int_{T_0} x(t)\, dt.$

Example:

$$\langle \cos(2\pi f_c t + \theta) \rangle = \frac{1}{T_0} \int_{T_0} \cos(2\pi f_c t + \theta)\, dt = \frac{1}{T_0} \times 0 = 0$$

Period $T_0 = \frac{1}{f_c}$



$$\langle \sin(2\pi f_c t + \theta) \rangle = 0$$

$$\langle \cos^2(2\pi f_c t + \theta) \rangle = \langle \tfrac{1}{2}(1 + \cos(2\pi 2 f_c t + 2\theta)) \rangle = \frac{1}{2}$$

Three Important Facts

(i) • If $g(t)$ is periodic with period T_0, then $P_g = \frac{1}{T_0}\int_{T_0} |g(t)|^2 dt$

(ii) • If $g(t)$ can be expressed as $\sum_{k=1}^{n} c_k e^{j2\pi f_k t}$ where the f_k are distinct, then $P_g = \sum_{k=1}^{n} |c_k|^2$.

- Ex

$$g(t) = A e^{j(2\pi f_c t + \theta)} \Rightarrow P_g = |A|^2$$

$$g(t) = A \cos(2\pi f_c t + \theta) \Rightarrow P_g = \frac{|A|^2}{2}$$

When $g(t)$ is periodic with period T_0:

$$g(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi \frac{k}{T_0} t} \Rightarrow P_g = \sum_{k=-\infty}^{\infty} |c_k|^2$$

(iii) • Suppose that $g(t)$ can be expressed as $a(t)\cos(2\pi f_c t + \phi)$ and that $A(f - f_c)$ and $A(f + f_c)$ do not overlap. Then

$$P_g = \frac{1}{2} P_a$$

Ex $g(t) = 1e^{j2\pi f_0 t} \Rightarrow P_g = 1^2 = 1$

$$P_g = \langle |g(t)|^2 \rangle = \langle |e^{j2\pi f_0 t}|^2 \rangle = \langle 1 \rangle = 1$$

$g(t) = e^{j2\pi 3t} + e^{j2\pi 4t} \Rightarrow P_g = 1^2 + 1^2 = 2$

$g(t) = e^{j2\pi 3t} + e^{j2\pi 3t} \Rightarrow P_g = 2^2 = 4$

$= 2e^{j2\pi 3t}$ $\quad \neq 1^2 + 1^2$

$g(t) = Ae^{j(2\pi f_c t + \theta)} \Rightarrow P_g = |Ae^{j\theta}|^2$

$= Ae^{j\theta} e^{j2\pi f_c t}$ $\quad = |A|^2$

$$g(t) = \cos(2\pi f_c t) = \frac{1}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right) \Rightarrow P_g = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 2 \times \frac{1}{4} = \frac{1}{2}$$

Alternatively, $P_g = \langle \cos^2(2\pi f_c t) \rangle = \frac{1}{2}$

$$g(t) = A\cos(2\pi f_c t + \theta)$$

$$= \frac{A}{2}\left(e^{j(2\pi f_c t + \theta)} + e^{-j(2\pi f_c t + \theta)}\right)$$

$$\Rightarrow P_g = \left|\frac{A}{2}\right|^2 + \left|\frac{A}{2}\right|^2 = \frac{|A|^2}{2}$$

(iii) $x(t) = \underbrace{m(t)}\sqrt{2}\cos(2\pi f_c t) \Rightarrow P_x = \frac{1}{2} \underbrace{P_{\sqrt{2}m(t)}} = \frac{1}{2} \times 2 P_m = P_m$

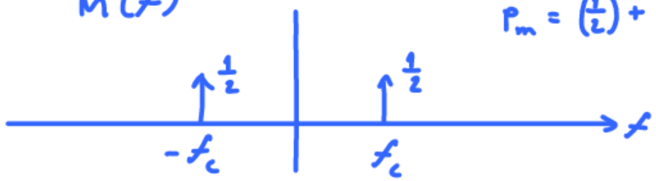
shifted copies do not overlap

$\langle (c\, m(t))^2 \rangle = \langle c^2 m^2(t) \rangle = c^2 \langle m^2(t) \rangle$

When they overlap:

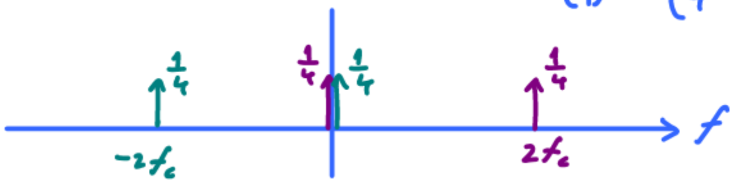
$m(t) = \cos(2\pi f_c t) \Rightarrow P_m = \frac{1}{2}$

M(f)



$P_m = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$

$m(t)\cos(2\pi f_c t) \Rightarrow P = \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4} + \frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{3}{8} \neq \frac{P_m}{2}$



"Proof" of (iii)

A(f)

$P_a = |a_1|^2 + |a_2|^2 + \cdots + |a_6|^2$

$g(t) = a(t)\cos(2\pi f_c t)$

$P_g = \left(\left|\frac{a_1}{2}\right|^2 + \left|\frac{a_2}{2}\right|^2 + \cdots + \left|\frac{a_6}{2}\right|^2 \right) \times 2$

$= \underbrace{\left(|a_1|^2 + |a_2|^2 + \cdots + |a_6|^2 \right)}_{P_a} \times \frac{1}{2}$

Power Calculation (More Examples)

non-overlapping → "$P_\Sigma = \Sigma P$"

(iv) $$g(t) = \sum_k a_k(t)\cos(2\pi f_k t + \phi_k)$$

$$P_g = \frac{1}{2}\sum_k P_{a_k} = \sum_k \frac{P_{a_k}}{2}$$

Assume the $A_k(f \pm f_k)$'s do not overlap.

(v) $$g(t) = a_1\cos(2\pi f_c t + \phi_1) + a_2\cos(2\pi f_c t + \phi_2)$$

(f_c … same … f_c)

$$P_g = \frac{1}{2}\left|a_1 e^{j\phi_1} + a_2 e^{j\phi_2}\right|^2$$

$$= \frac{1}{2}a_1^2 + \frac{1}{2}a_2^2 + a_1 a_2 \cos(\phi_2 - \phi_1)$$

Ex. $g(t) = 4\cos(2t) + 3\sin(2t)$

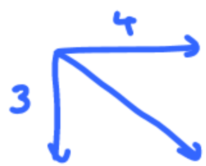
$= 4\cos(2t) + 3\cos(2t - 90°)$

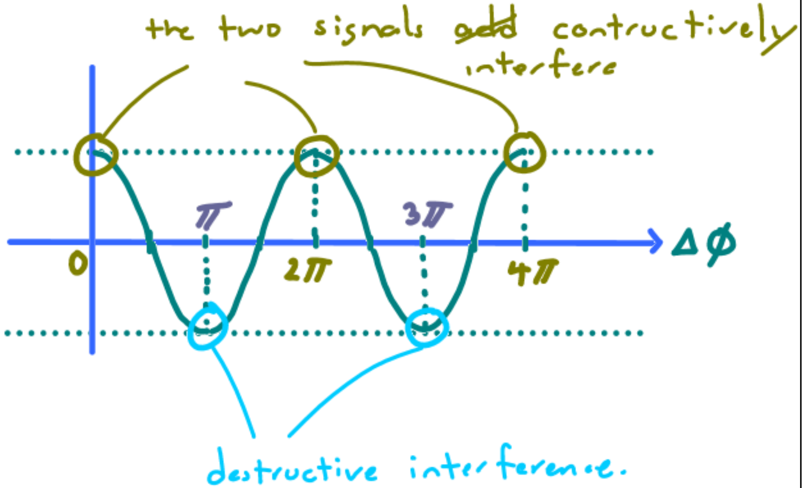
$\equiv 4\angle 0° + 3\angle -90°$

$= 5\angle -36.87°$

$= 5\cos(2t - 36.87°)$

$P_g = \frac{5^2}{2}$





$(a_1 - a_2)^2 \geq 0$

$a_1^2 + a_2^2 - 2a_1a_2 \geq 0$

$\frac{a_1^2}{2} + \frac{a_2^2}{2} \geq a_1 a_2$

$$\cos x = \frac{e^{+jx} + e^{-jx}}{2} = \operatorname{Re}\{e^{jx}\}$$

$$g(t) = a_1 \cos(2\pi f_c t + \phi_1) + a_2 \cos(2\pi f_c t + \phi_2)$$

$$= \operatorname{Re}\{a_1 e^{j(2\pi f_c t + \phi_1)}\} + \operatorname{Re}\{a_2 e^{j(2\pi f_c t + \phi_2)}\}$$

$$= \operatorname{Re}\{\underbrace{(a_1 e^{j\phi_1} + a_2 e^{j\phi_2})}_{Z} e^{j(2\pi f_c t)}\}$$

$$= \operatorname{Re}\{a e^{j\phi} e^{j 2\pi f_c t}\} = \operatorname{Re}\{a e^{j(2\pi f_c t + \phi)}\}$$

$$= a \cos(2\pi f_c t + \phi)$$

$$z = a e^{j\phi}$$

$$P_g = \frac{a^2}{2}$$

$$|z|^2 = z z^* = (a_1 e^{j\phi_1} + a_2 e^{j\phi_2})(a_1 e^{-j\phi_1} + a_2 e^{-j\phi_2})$$

$$= a_1^2 + a_2^2 + a_1 a_2 e^{j(\phi_1 - \phi_2)} + a_1 a_2 e^{-j(\phi_1 - \phi_2)}$$

$$= a_1^2 + a_2^2 + 2\cos(\phi_1 - \phi_2)\, a_1 a_2$$

Ray tracing (a revisit)

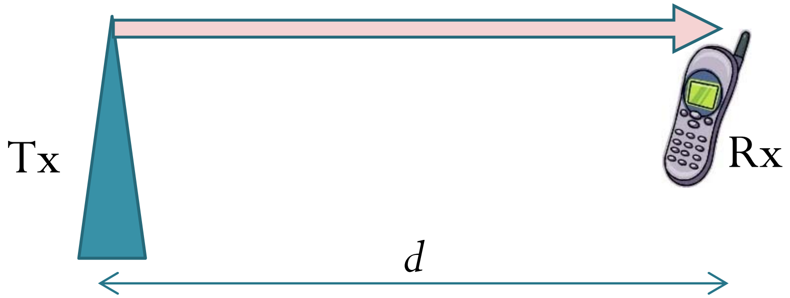
$$P_r = \left(\frac{\sqrt{G_{Tx}G_{Rx}}\,\lambda}{4\pi d}\right)^2 P_t = \left(\frac{\alpha}{d}\right)^2 P_t$$

- LOS:

$$P_{rx} = \frac{\left(\sqrt{2P_t}\right)^2}{2} = P_t$$

$$x(t) = \sqrt{2P_t}\cos(2\pi f_c t)$$

$$y(t) = \frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{d}{c}\right)\right)$$



From Friis equation,

$$\alpha = \frac{\sqrt{G_{Tx}G_{Rx}}\lambda}{4\pi}.$$

- Multipath Reception

$$y(t) = \sum_{k=1}^{n} R_k \frac{\alpha}{r_k}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_k}{c}\right)\right)$$

R_k ← reflection coefficient

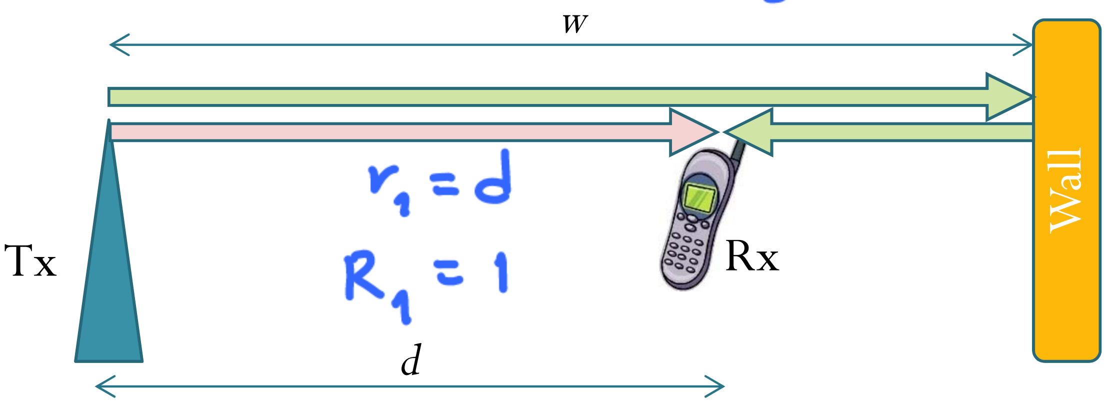
assume $= \begin{cases} 1 & \text{for no reflection} \\ -1 & \text{for one reflection} \end{cases}$

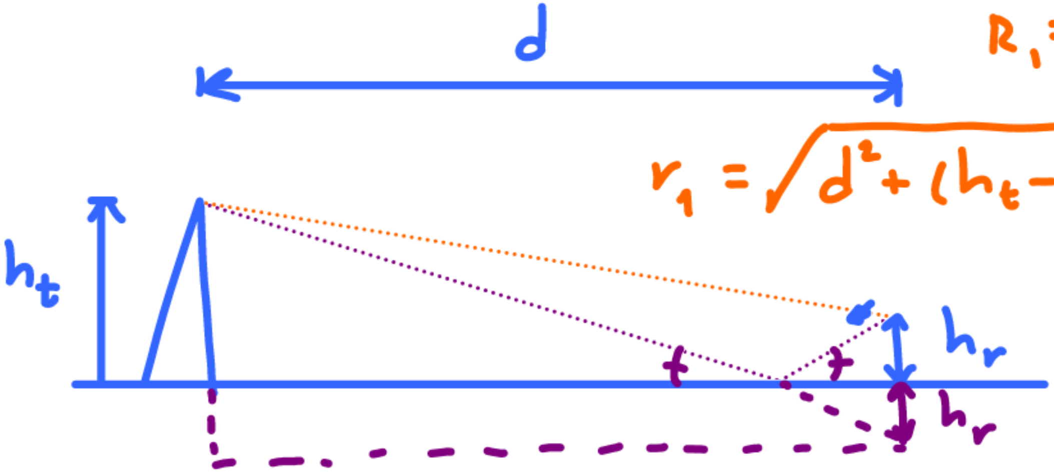
Ex. One reflecting wall (1/4)

- There is a fixed antenna transmitting the sinusoid $x(t)$, a fixed receive antenna, and a single perfectly reflecting large fixed wall.
- Assume that the wall is very large, the reflected wave at a given point is the same (except for a sign change) as the free space wave that would exist on the opposite side of the wall if the wall were not present

$R_2 = -1$

$r_2 = w + (w - d) = 2w - d$





$R_1 = 1$

$r_1 = \sqrt{d^2 + (h_t - h_r)^2}$

$r_2 = \sqrt{d^2 + (h_t + h_r)^2}$

$R_2 = -1$

Ex. One reflecting wall (2/4)

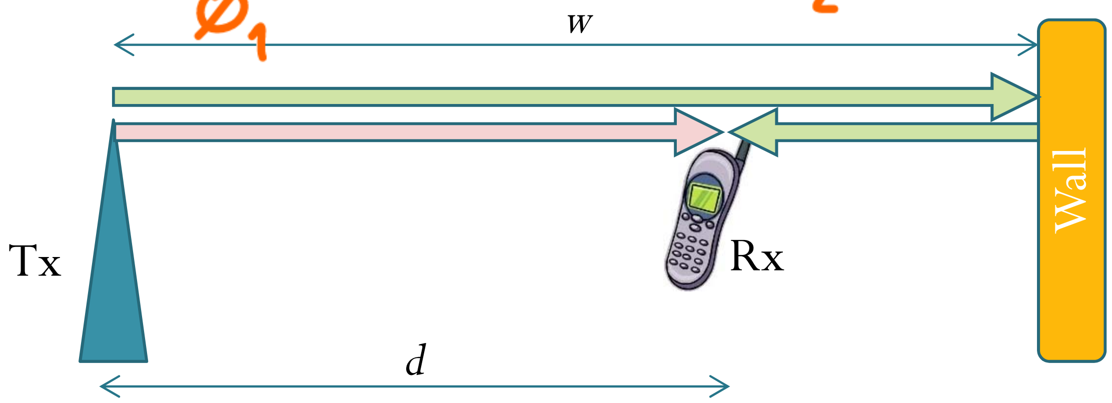
$$x(t)=\sqrt{2P_t}\cos(2\pi f_c t)$$

$$y(t)=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)-\frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)\right)$$

$$=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)-\frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)\right)$$

$$=\frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right)+\frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)-\pi\right)$$

a_1 $\quad\phi_1$ $\quad a_2$ $\quad\phi_2$



Ex. One reflecting wall (3/4)

$$y(t) = \frac{\alpha}{d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{d}{c}\right)\right) + \frac{\alpha}{2w-d}\sqrt{2P_t}\cos\left(2\pi f_c\left(t-\frac{2w-d}{c}\right)-\pi\right)$$

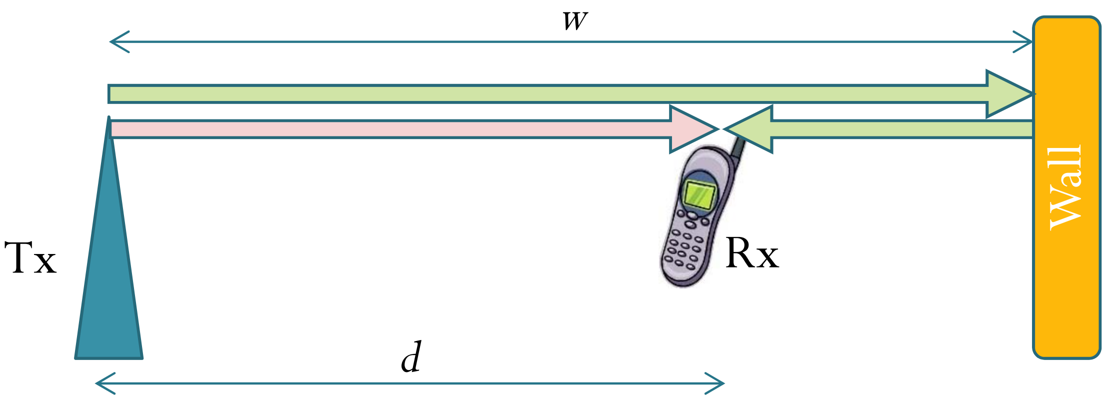
$$P_y = P_t\left(\left(\frac{\alpha}{d}\right)^2 + \left(\frac{\alpha}{2w-d}\right)^2 + 2\frac{\alpha^2}{d(2w-d)}\cos(\Delta\phi)\right)$$

$$\Delta\phi = 2\pi f_c \frac{2w-2d}{c} + \pi = 2\pi\frac{1}{\lambda/2}(w-d) + \pi$$

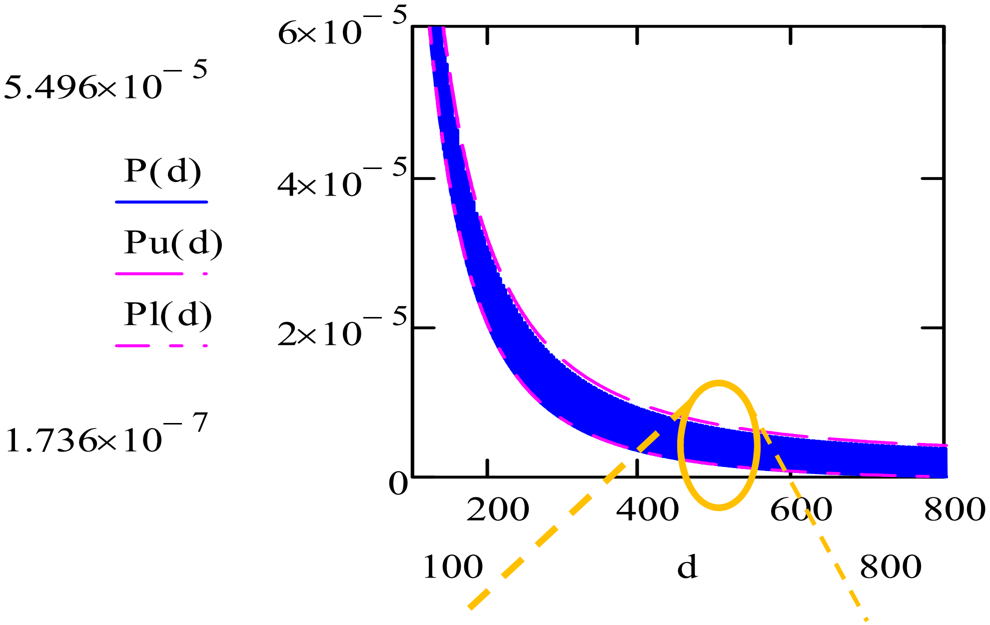
$\cos(2\pi 5x)$

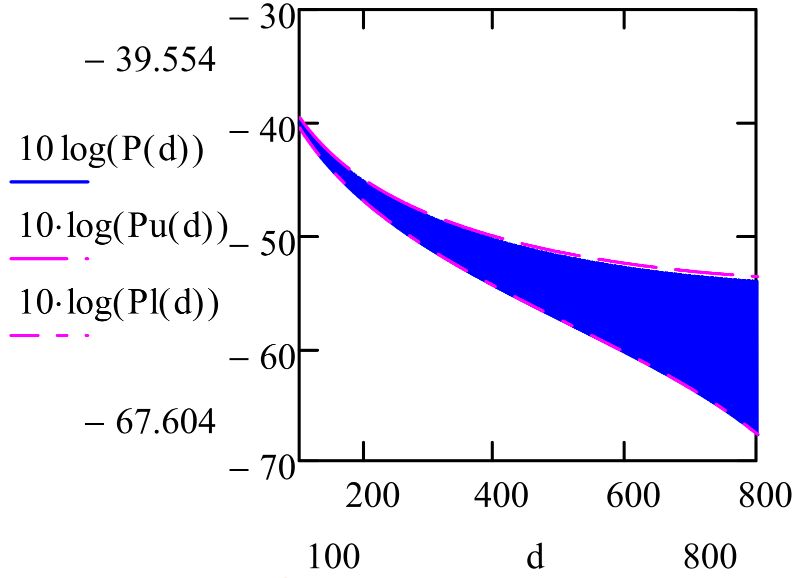
period $= \frac{\lambda}{2}$

form constructive and destructive interference pattern

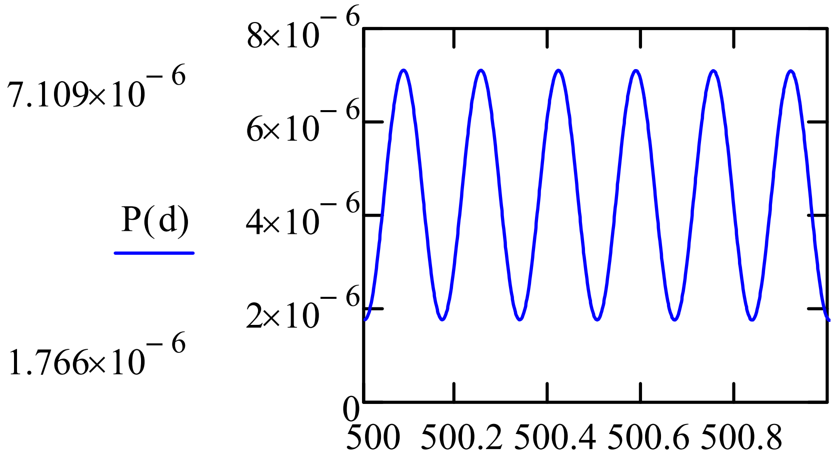


Ex. One reflecting wall (4/4)





$$\text{period} = \frac{1}{6} \approx 0.167 \text{ m}$$



$$f = 900 \text{ MHz}$$

$$w = 1 \text{ km}$$

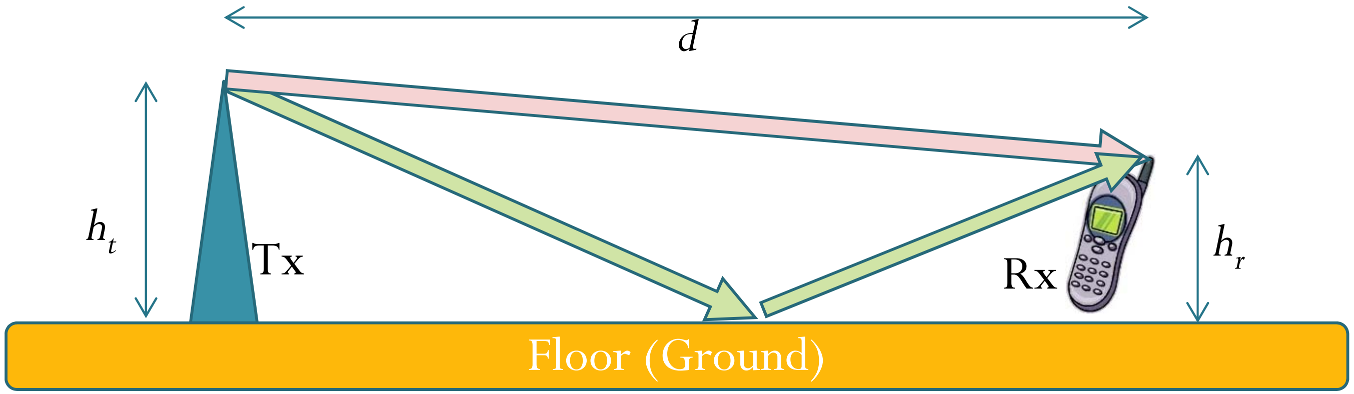
$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^8} = \frac{1}{3}$$

Ex. Two-Ray Model

$$\text{Delay spread} = \frac{r_2}{c} - \frac{r_1}{c}$$

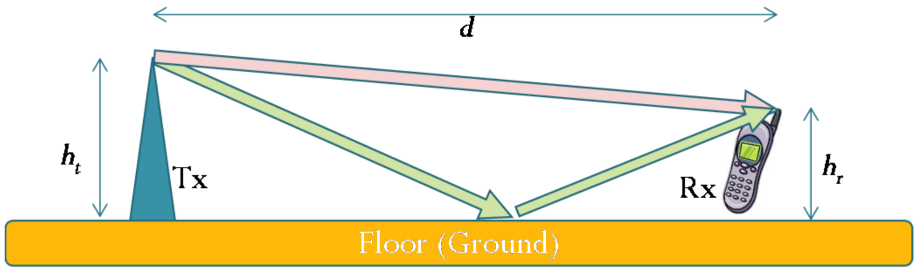
$$y(t) = \frac{\alpha}{r_1}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_1}{c}\right)\right) - \frac{\alpha}{r_2}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r_2}{c}\right)\right)$$

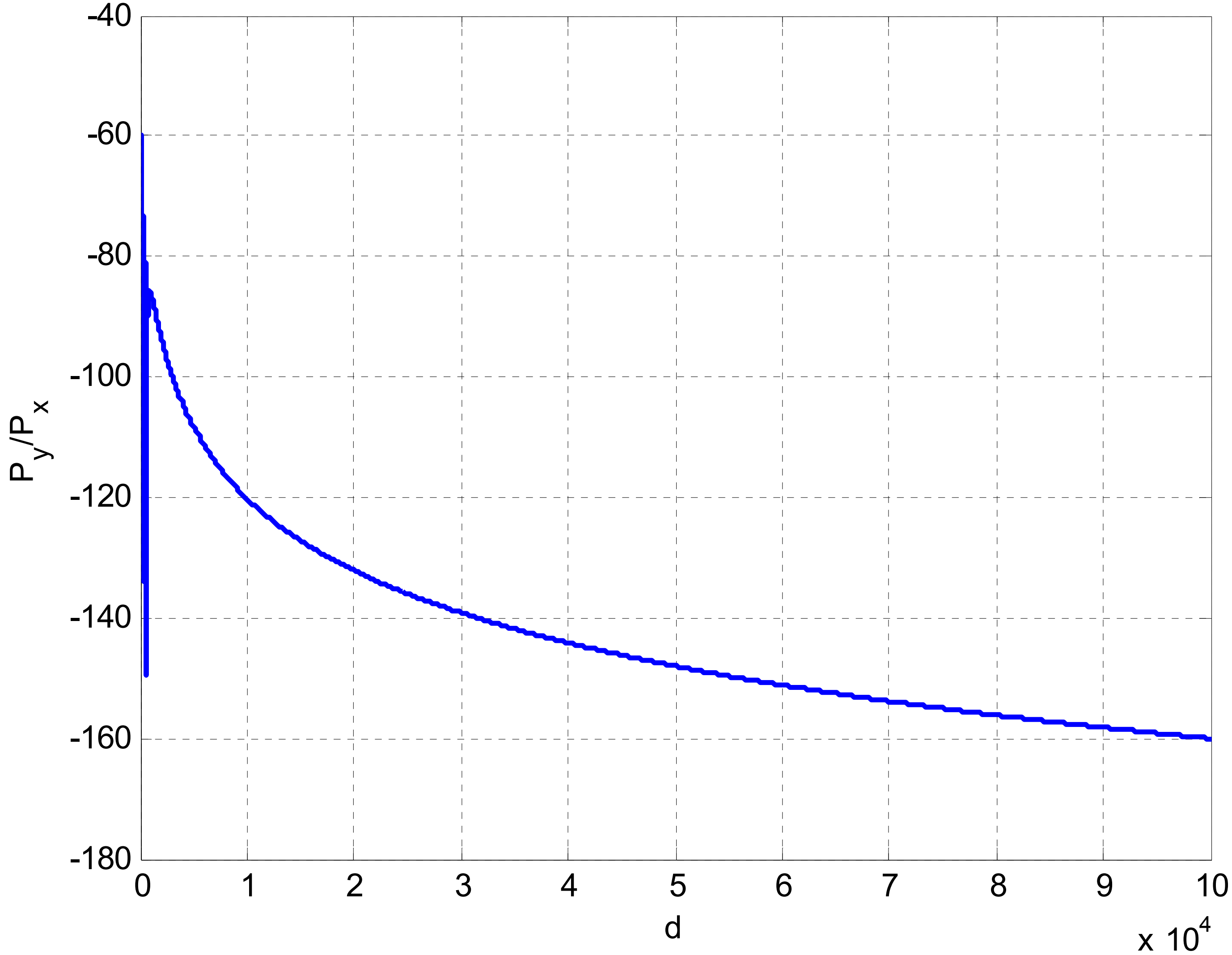
$$\frac{P_y}{P_x} = \left|\frac{\alpha}{r_1}e^{-j2\pi f_c \frac{r_1}{c}} - \frac{\alpha}{r_2}e^{-j2\pi f_c \frac{r_2}{c}}\right|^2 = \left|\frac{\alpha}{r_1} - \frac{\alpha}{r_2}e^{-j2\pi f_c \frac{r_2 - r_1}{c}}\right|^2$$



Assume ground reflection coefficient = -1.

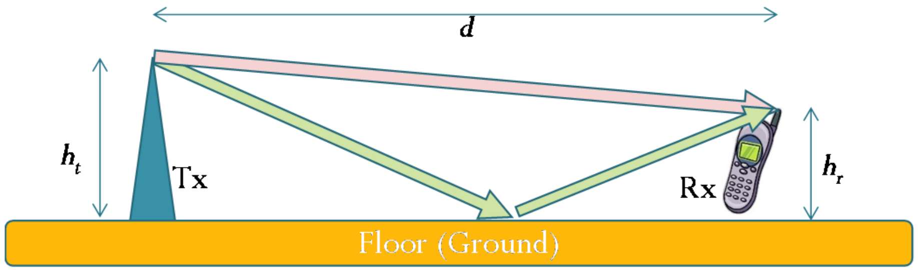
Ex. Two-Ray Model

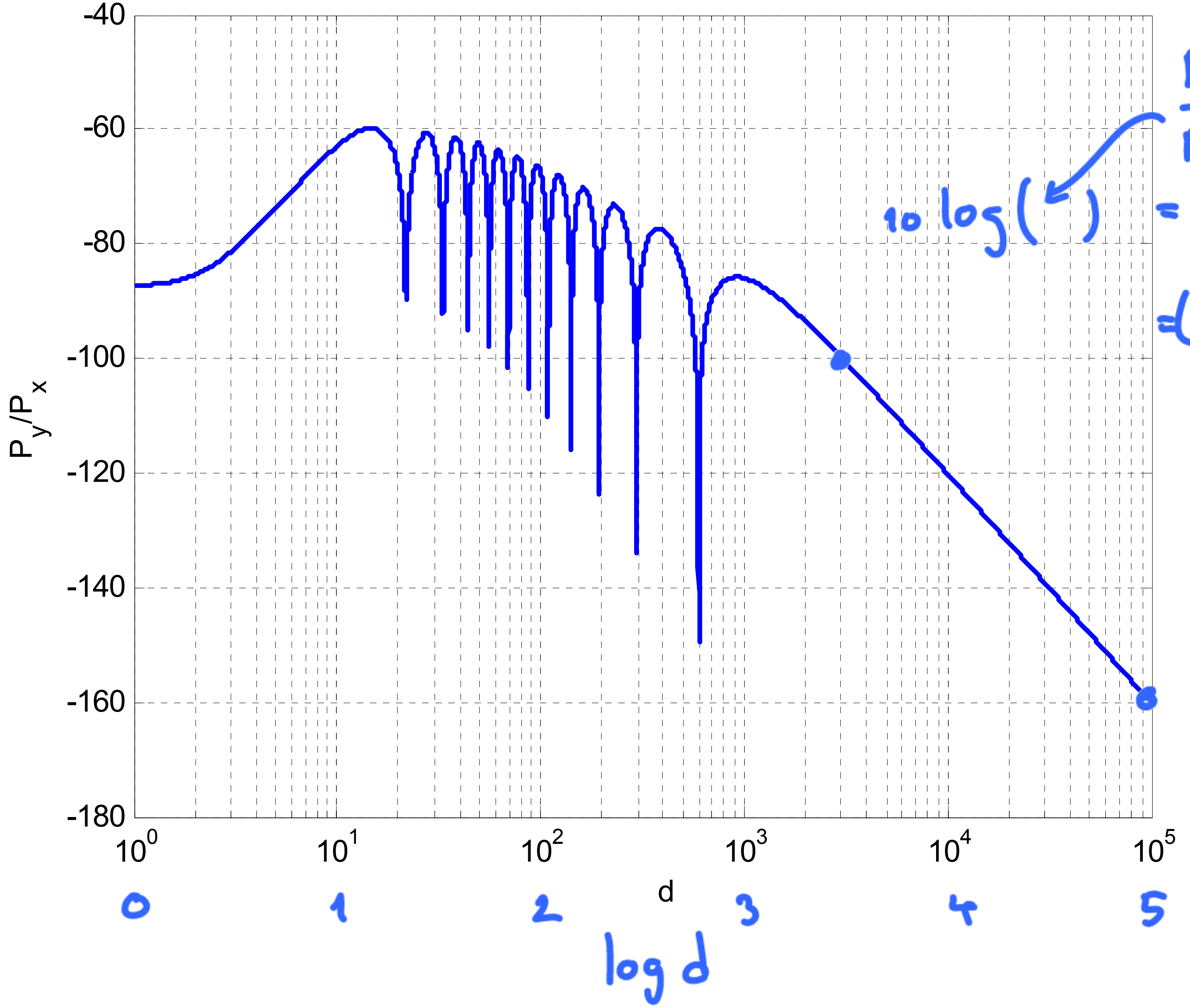




$$f = 900 \text{ MHz}$$
$$h_t = 50 \text{ m}$$
$$h_r = 2 \text{ m}$$

Ex. Two-Ray Model





$$\frac{P_r}{P_t} = K\left(\frac{d_0}{d}\right)^{\gamma}$$

$$10\log(\;) = 10\log K + \gamma 10 \log\frac{d_0}{d}$$

$$= (\;) - 10\gamma\log d$$

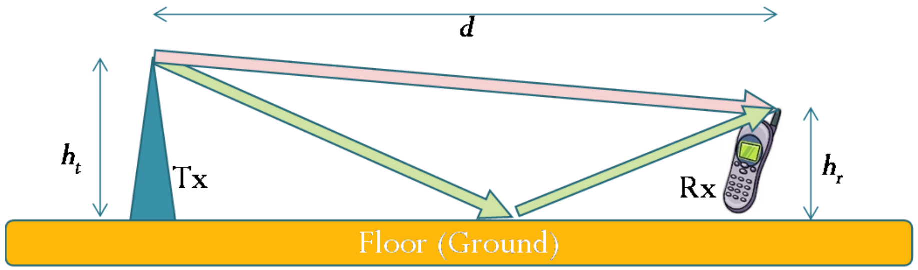
$f = 900$ MHz

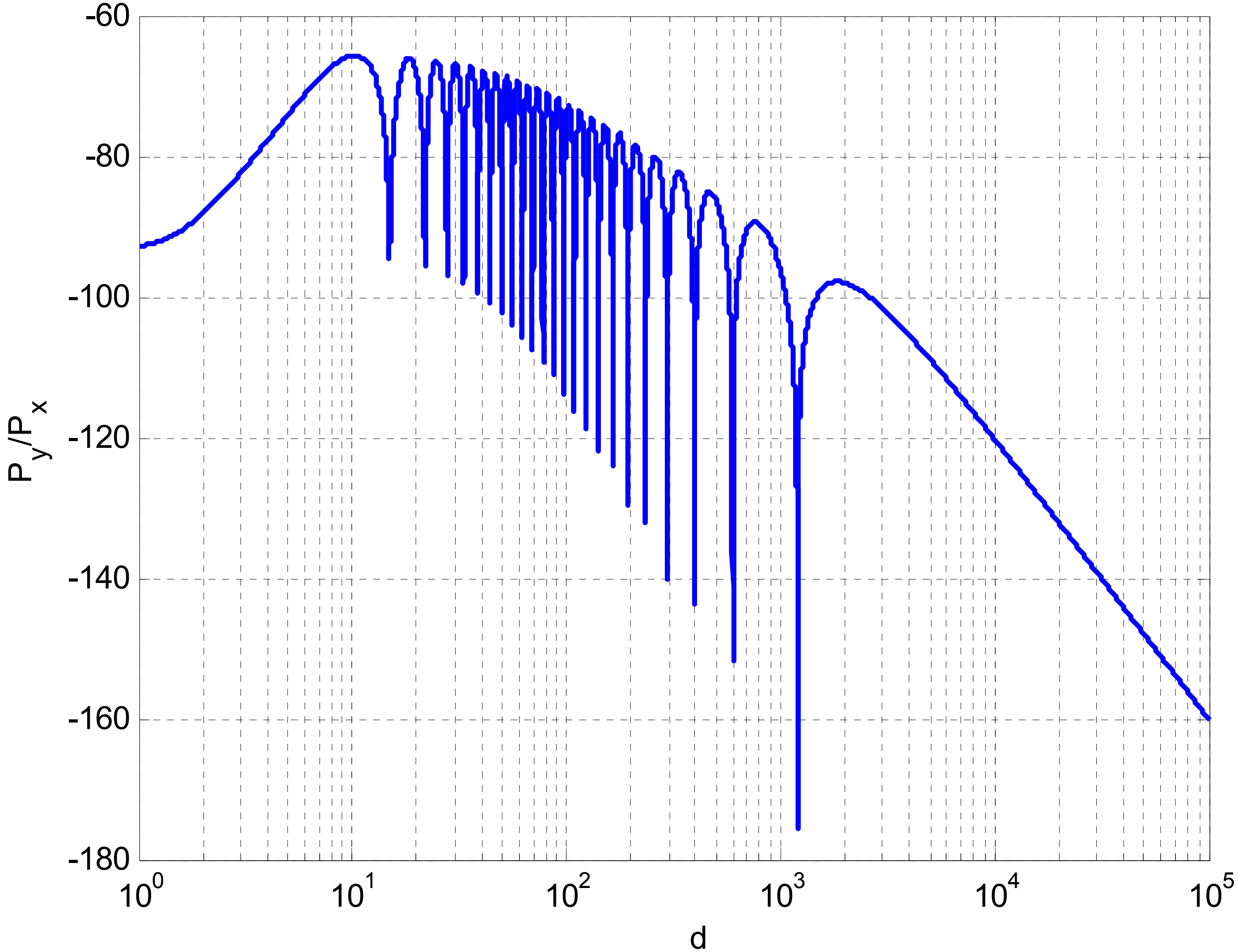
$h_t = 50$ m

$h_r = 2$ m

slope ≈ -40

Ex. Two-Ray Model





$$f = 1800 \text{ MHz}$$

$$h_t = 50 \text{ m}$$

$$h_r = 2 \text{ m}$$

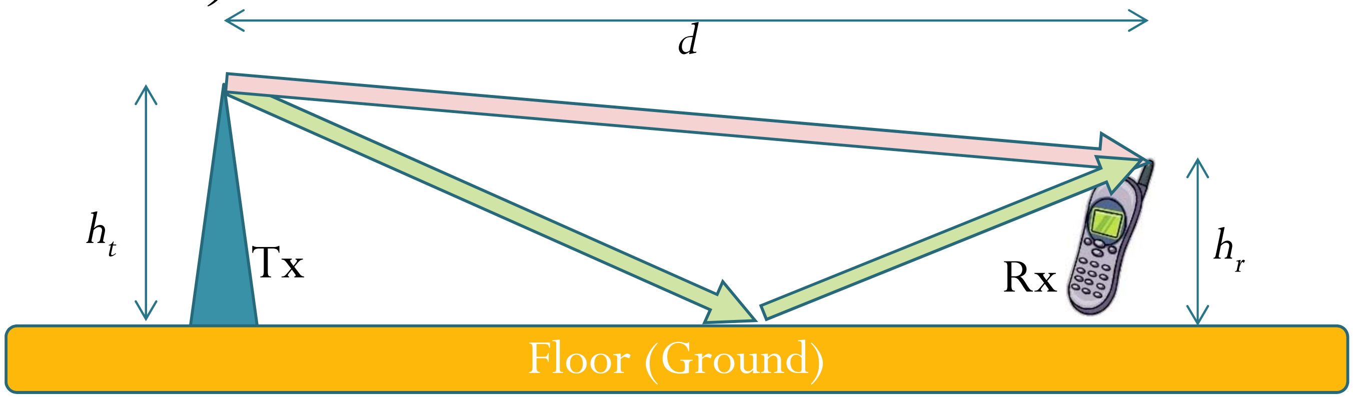
Ex. Two-Ray Model (Approximation)

$$d \gg h_t, h_r$$

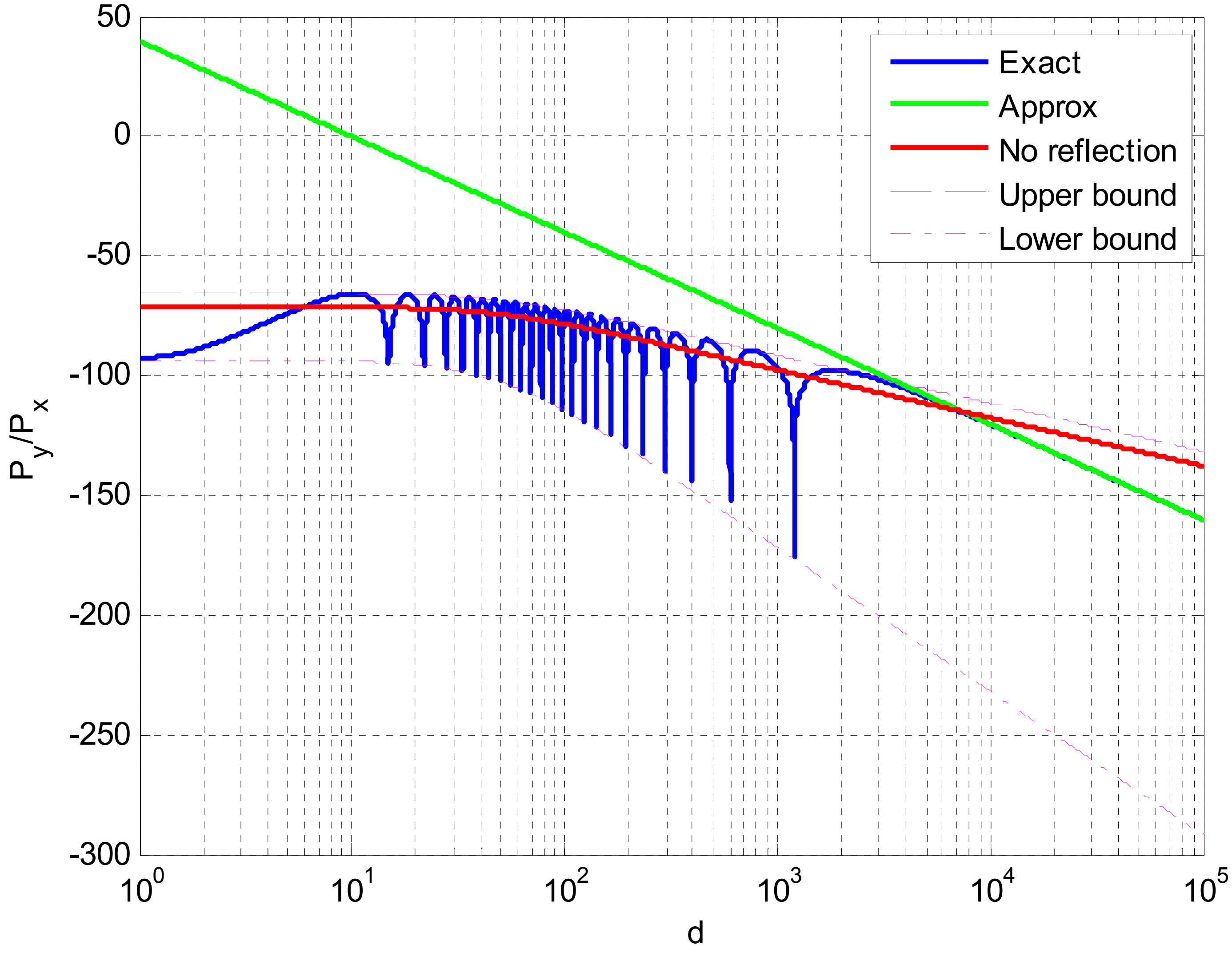
$$\frac{P_y}{P_x} \approx \left| \frac{\alpha}{r_1} - \frac{\alpha}{r_2} e^{-j2\pi \frac{2h_t h_r}{\lambda}} \right|^2 \approx \frac{\alpha}{d} \left| 1 - e^{-j2\pi \frac{2h_t h_r}{\lambda d}} \right|^2$$

$$\approx \left(\frac{\alpha}{d}\right)^2 \left| 1 - \left(1 - j2\pi \frac{2h_t h_r}{\lambda d}\right) \right|^2 = \frac{\alpha^2}{d^2} \left| j2\pi \frac{2h_t h_r}{\lambda d} \right|^2 = \left(\frac{4\pi\alpha h_t h_r}{\lambda d^2} \right)^2$$

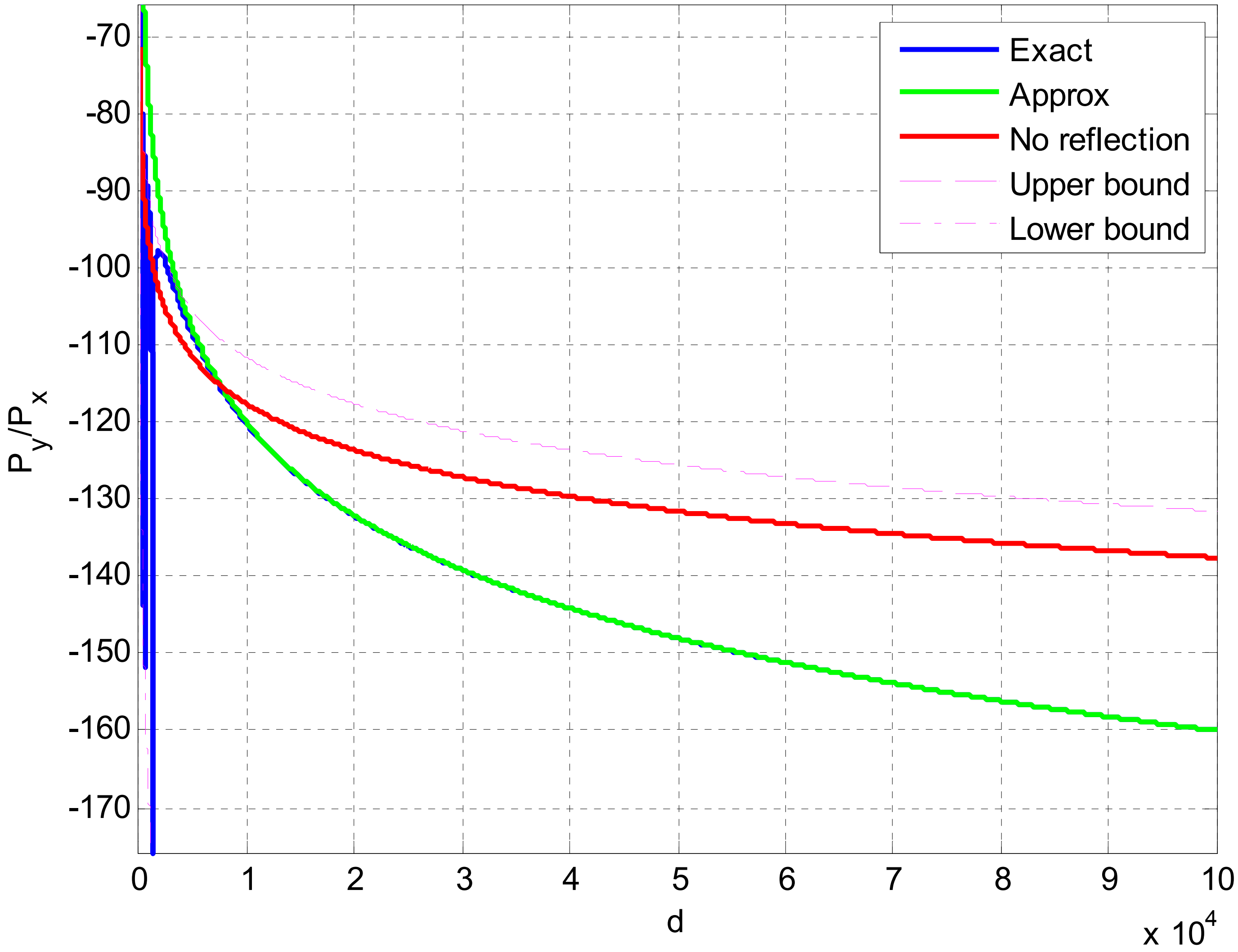
$$= \left(\frac{\sqrt{G_{Tx} G_{Rx}} h_t h_r}{d^2} \right)^2 \propto \frac{1}{d^4}$$



Ex. Two-Ray Model



Ex. Two-Ray Model



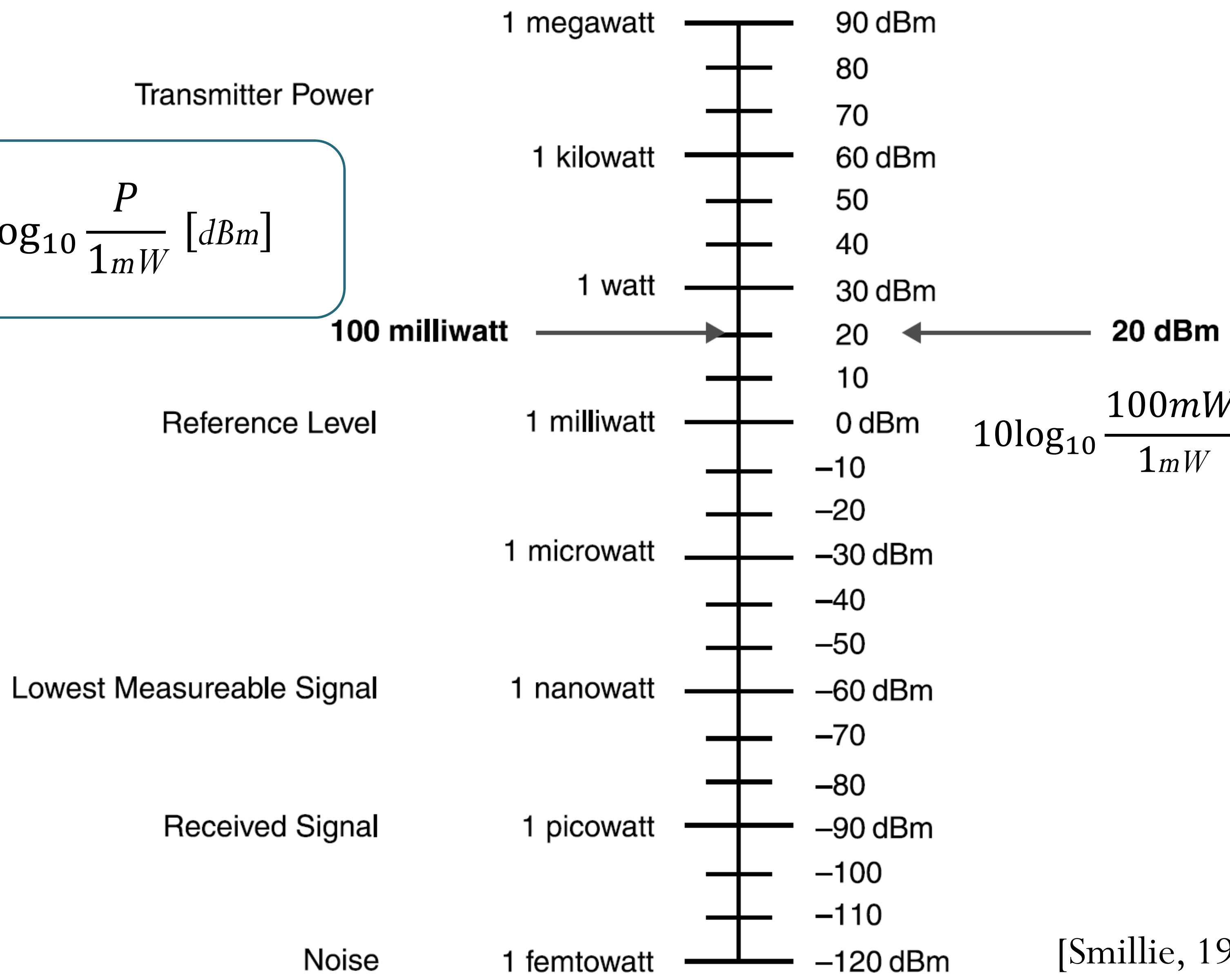
dBm

- The range of RF power that must be measured in cellular phones and wireless data transmission equipment varies from
  - hundreds of watts in base station transmitters to
  - picowatts in receivers.
- For calculations to be made, all powers must be expressed in the same power units, which is usually **milliwatts**.
  - A transmitter power of 100 W is therefore expressed as 100,000mW. A received power level of 1 pW is therefore expressed as 0.000000001mW.
- Making power calculations using decimal arithmetic is therefore complicated.
- To solve this problem, the dBm system is used.

[Scott and Frobenius, 2008, Fig 1.1]

Range of RF Power in Watts and dBm

$$P\,[W] = 10\log_{10}\frac{P}{1mW}\,[dBm]$$



$$10\log_{10}\frac{100mW}{1mW} = 20\,[dBm]$$

[Smillie, 1999, p 11]

dB and dBm

- The decibel scale expresses factors or ratios logarithmically.
- Unitless dB value
  - Represent power ratio: $10\log_{10}\frac{P_2}{P_1}$
- dB value with a unit
  - Represent the signal power itself:

$$P[\text{dBW}] = 10\log_{10}\frac{P}{1\ \text{W}}, \qquad P[\text{dBm}] = 10\log_{10}\frac{P}{1\ \text{mW}}$$

  - Note that $P[\text{dBm}] = P[\text{dBW}] + 30$

Remark

- Adding dB values corresponds to multiplying the underlying factors, which means multiplying the units if they are present.
- It is therefore appropriate to add unitless dB values to a dB value with a unit (such as dBm)
  - The result is still referred to that unit.
  - Ex: 17 dBm + 13 dB – 6 dB = 24 dBm
    - Correspond to 50 mW × 20 / 4 = 250 mW.

Doppler Shift: 1D Move

- At the transmitter, suppose we have

$$\sqrt{2P_t}\cos(2\pi f_c t + \phi)$$

- At distance *r* (far enough), we have

Time to travel a distance of *r*

$$\frac{\alpha}{r}\sqrt{2P_t}\cos\left(2\pi f_c\left(t - \frac{r}{c}\right) + \phi\right)$$

- If moving, *r* becomes *r*(*t*).
- If moving ***away*** at a constant velocity *v*, then $r(t) = r_0 + vt.$

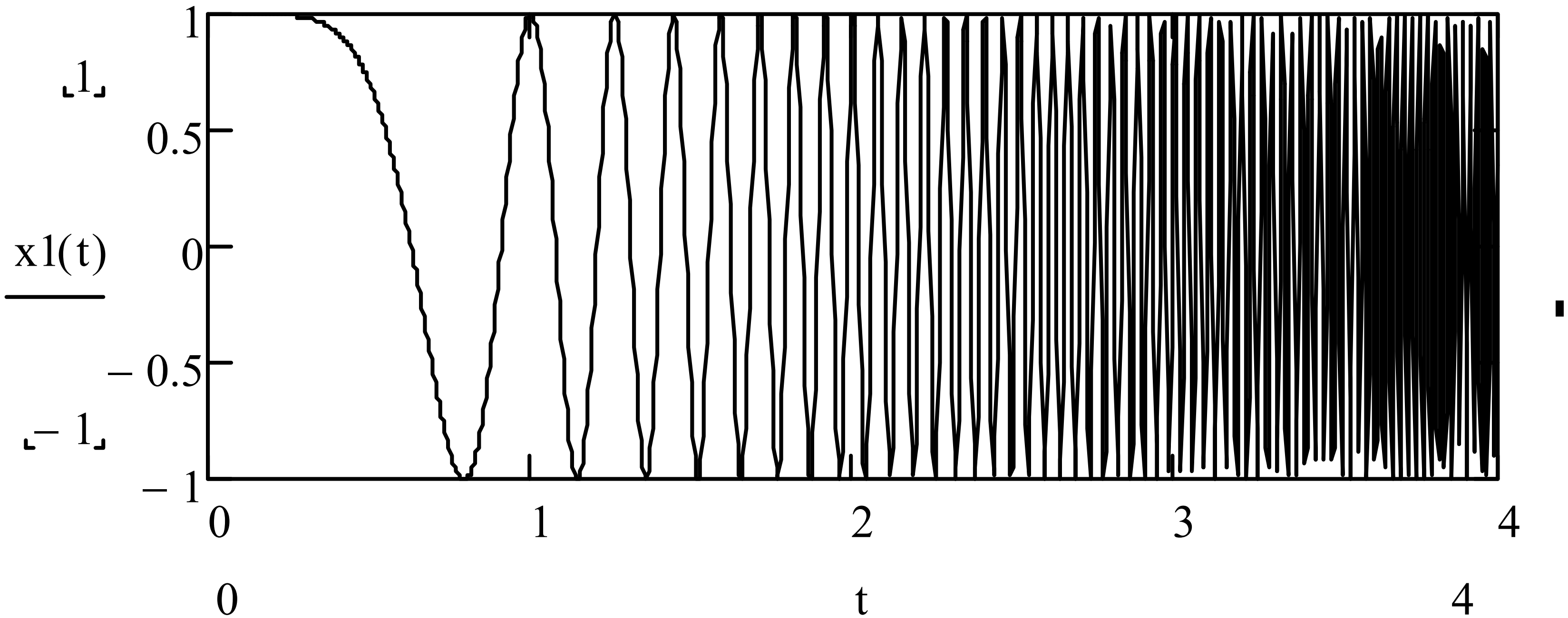
$$\frac{\alpha}{r(t)}\cos\left(2\pi f_c\left(t - \frac{r_0 + vt}{c}\right) + \phi\right) = \frac{\alpha}{r(t)}\cos\left(2\pi\left(f_c - f_c\frac{v}{c}\right)t - 2\pi f_c\frac{r_0}{c} + \phi\right)$$

Frequency shift

$$\Delta f = \frac{v}{\lambda}$$

Instantaneous Frquency

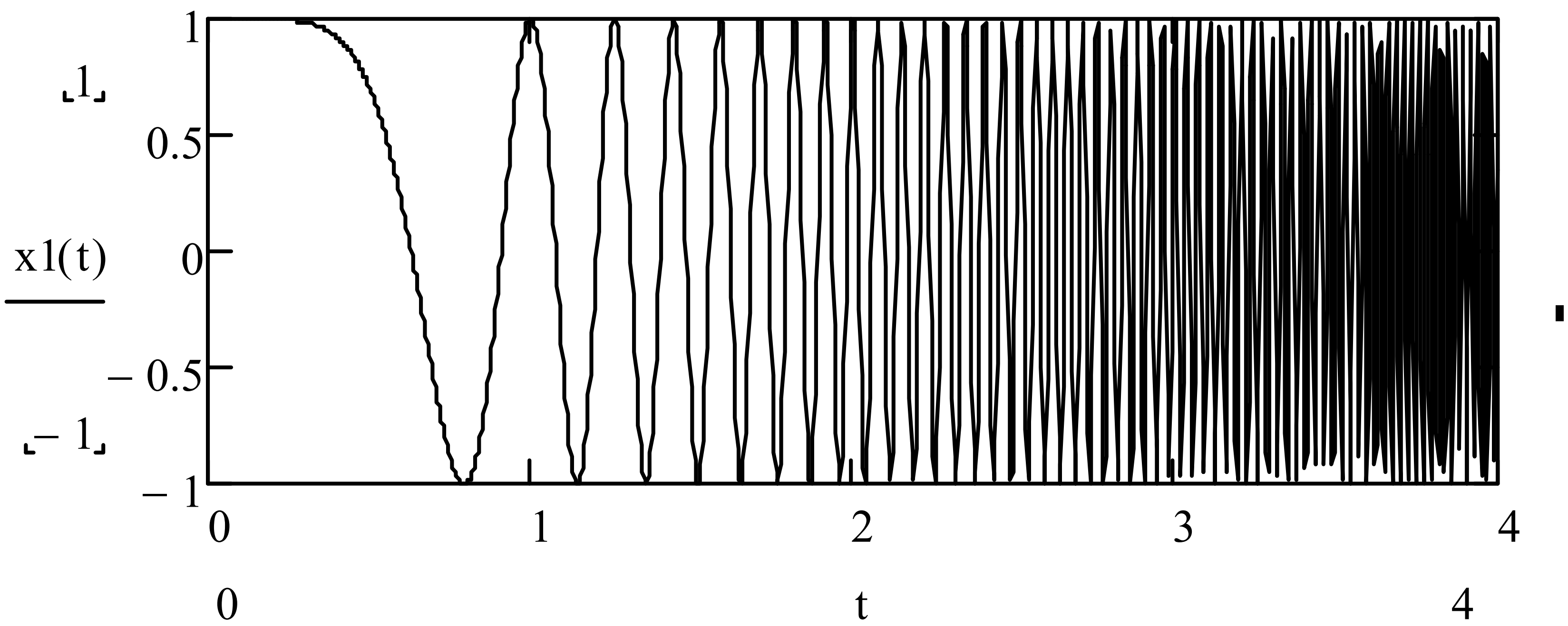
$$x_1(t) = \cos\left(2\pi t^2 t\right)$$



At t = 2, frequency = ?

Instantaneous Frquency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

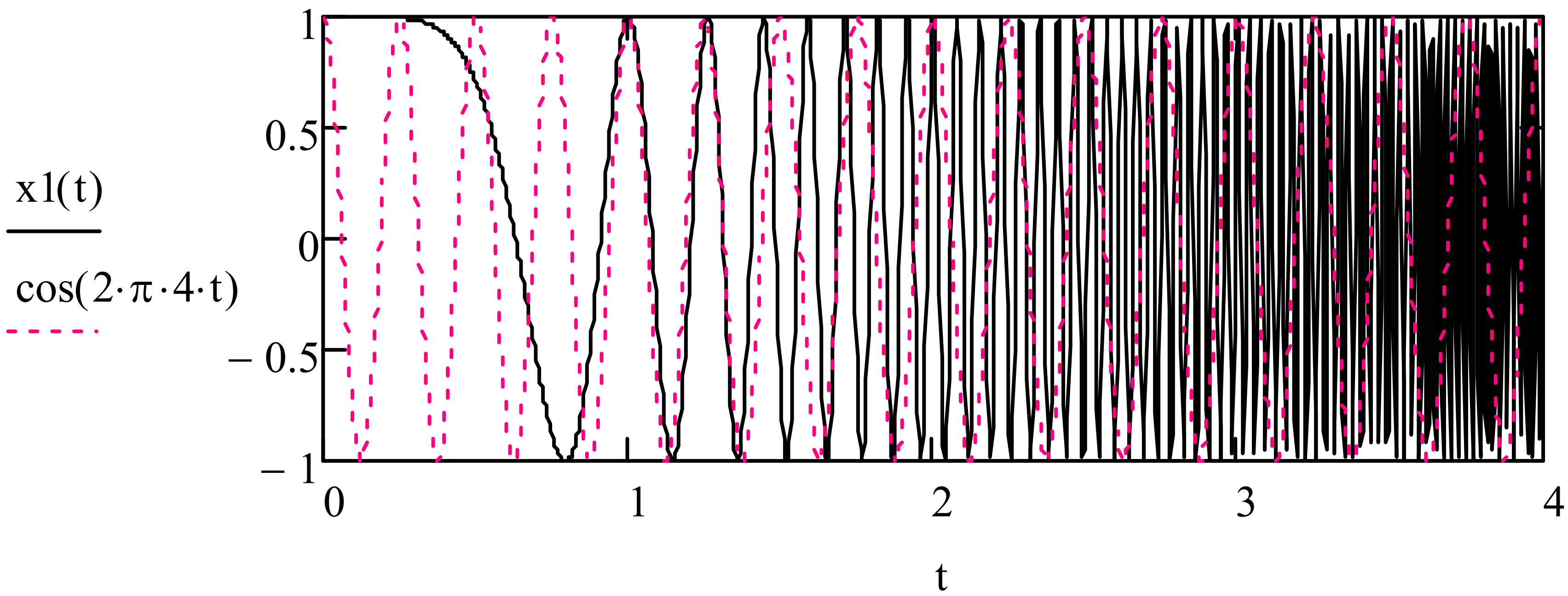


$\cos\left(2\pi ft\right)$ ➡ At t = 2, $f = t^2 = 4$ Hz?

Correct?

Instantaneous Frquency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

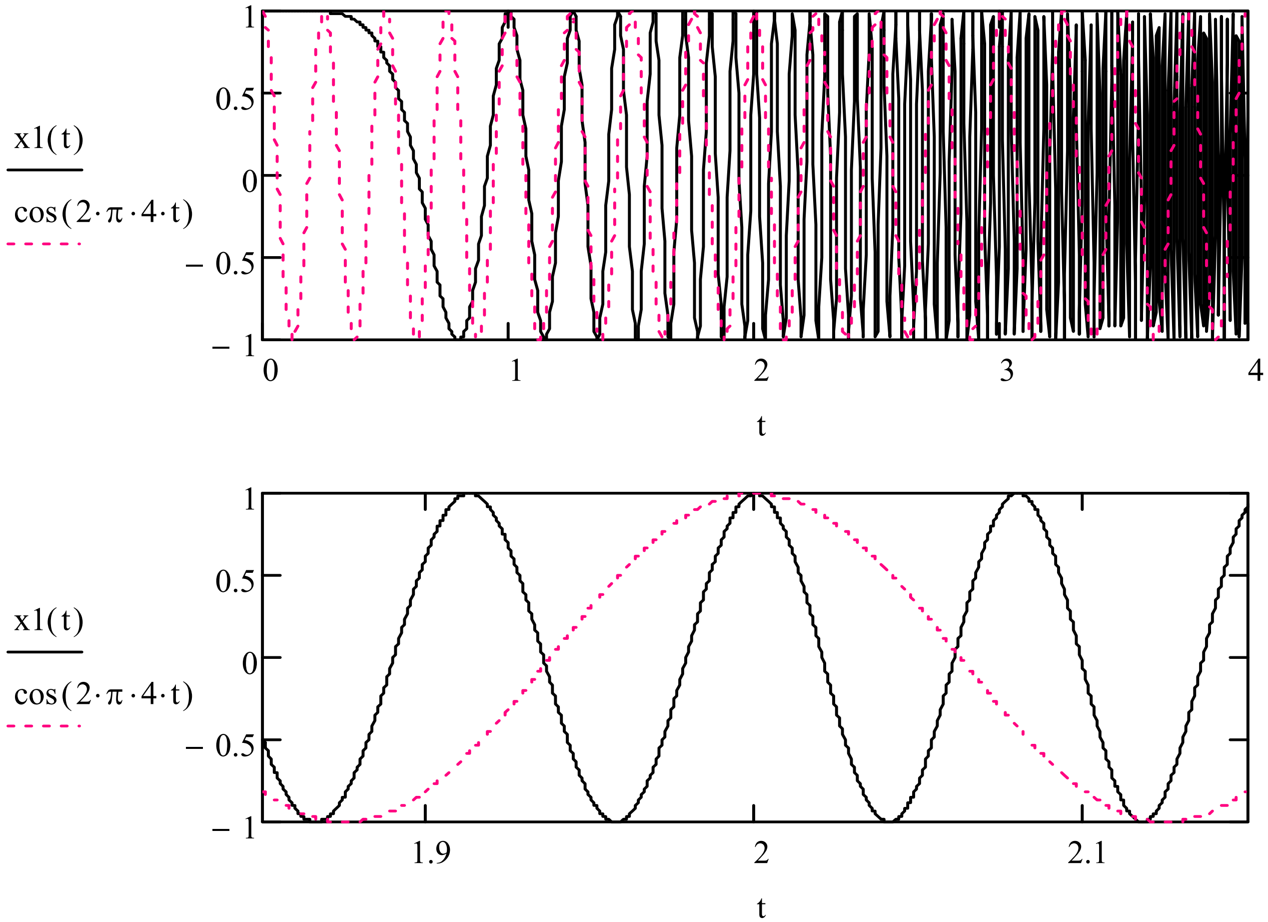


$\cos\left(2\pi ft\right)$ ➡ At t = 2, $f = t^2 = 4$ Hz?

Correct?

Instantaneous Frquency

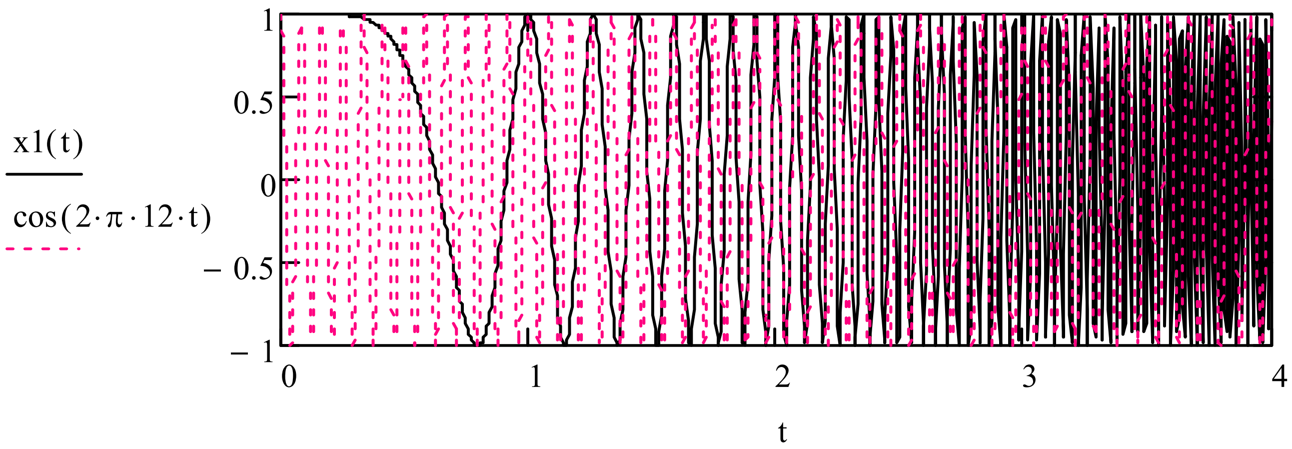
$$x_1(t) = \cos\left(2\pi t^2 t\right)$$

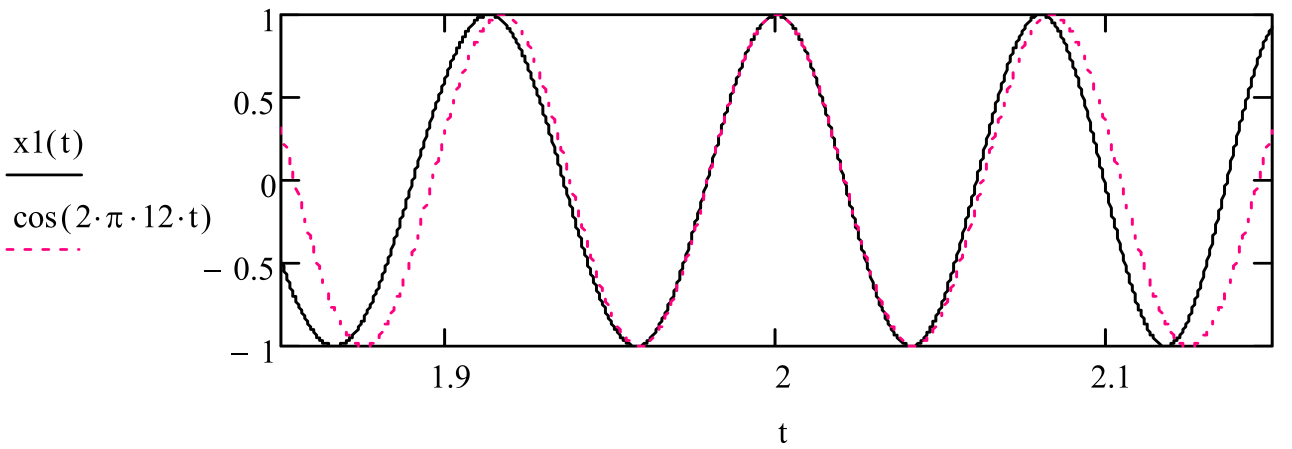


4 Hz is too low!!!

Instantaneous Frquency

$$x_1(t) = \cos\left(2\pi t^2 t\right)$$





12 Hz?

Review: Instantaneous Frequency

For a generalized sinusoid signal

$$A\cos\left(\theta(t)\right),$$

the **instantaneous frequency** at time t is given by

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t).$$

When $\theta(t) = 2\pi f_c\left(t - \frac{r(t)}{c}\right) + \phi,$

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t) = f_c - \frac{f_c}{c}\frac{d}{dt}r(t) = f_c - \frac{1}{\lambda}\frac{d}{dt}r(t)$$

Frequency shift

Big Picture

Transmission impairments in cellular systems

Physics of radio propagation	Attenuation (Path Loss) Shadowing Doppler shift Inter-symbol interference (ISI) Flat fading Frequency-selective fading
Extraneous signals	Co-channel interference Adjacent channel interference Impulse noise White noise
Transmitting and receiving equipment	White noise Nonlinear distortion Frequency and phase offset Timing errors

[Myung and Goodman, 2008, Table 2.1]